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A Lorentz invariant formulation of the Yang-Mills theory with gauge invariant ghost field Lagrangian

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Abstract: A new formulation of the Yang-Mills theory which allows a manifestly covariant gauge fixing accompanied by a gauge invariant ghost field interaction is proposed. The gauge condition selects a unique representative in the class of gauge equivalent configurations.

Keywords: Gauge Symmetry, Renormalization Regularization and Renormalons.

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Contents

1. Introduction

Quantization of the Yang-Mills theory involves fixing a gauge. A Lorentz invariant gauge fixing in non-Abelian theories requires introduction of additional anticommuting scalar fields, Faddeev-Popov ghosts [1], with non gauge invariant interaction. The ghost field interaction produces ultraviolet divergencies which has to be removed by renormalization of the ghost fields and the ghost-gluon interaction vertex. Moreover such a gauge fixing does not choose a unique representative in a class of gauge equivalent configurations. For large fields Gribov copies appear, which makes questionable using this procedure as a starting point for nonperturbative calculations. Contrary to the Quantum Electrodynamics (QED), where the quantization procedure does not break the conservation of the current interacting with the gauge field, in non-Abelian theories this conservation is broken both by the gauge fixing and by the presence of non-gauge invariant ghost field interaction. The Ward identities [2 − 4] which express in the quantum case the conservation of electromagnetic current are replaced in the non-Abelian theories by much more complicated relations, Slavnov-Taylor (ST) identities [5, 6], including the Green functions of composite operators. These relations may be interpreted as a consequence of the conservation of a more general current involving also the ghost fields [7 − 9].

Dynamical ghost fields are absent in the linear gauges, like $nA = 0$, which makes possible to obtain in this case relations between the Green functions, similar to the Ward identities in QED. However linear gauges break explicitly the Lorentz invariance, and their using is very cumbersome. According to the common wisdom in non-Abelian theories one has to sacrifice either explicit Lorentz invariance or gauge invariance of the ghost field action.

In this paper I propose a procedure which preserves in the quantum Yang-Mills theory simultaneously the manifest Lorentz invariance and the gauge invariance of the ghost field Lagrangian. It allows to obtain the relations between the Green functions following essentially the same procedure as in the Abelian case. The gauge fixing used in the present paper is free of Gribov ambiguity and in perturbation theory leads to the same results as the standard quantization procedure.

The paper is organized as follows. In the next section the model is described and its equivalence to the standard Yang-Mills theory is demonstrated. In the third section a new Lorentz invariant gauge condition free of Gribov ambiguity is introduced and the diagram technique is analyzed. The relations between the Green functions are derived. In conclusion I discuss the results obtained in this paper and their possible applications.

2. The model

In this section I consider the SU(2) gauge model. Generalization to other compact groups does not make serious problems.

We start with the usual path integral representation for the S-matrix in the Coulomb gauge

$$S = \int \exp\left\{ i \int [L_{\mathrm{YM}} + \lambda^a \partial_i A_i^a]\, dx \right\} d\mu \tag{2.1}$$

where

$$L_{\mathrm{YM}} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \tag{2.2}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\varepsilon^{abc} A_\mu^b A_\nu^c, \quad a, b, c, = 1, 2, 3 \tag{2.3}$$

The measure $d\mu$ includes differentials of all the fields as well as the Faddeev-Popov determinant $\det M$. This determinant is conveniently presented as the integral over anticommuting ghost fields

$$\det M = \int \exp\left\{ i \int \bar{c} \partial_i D_i c dx \right\} d\bar{c} dc \tag{2.4}$$

where D_i is a covariant derivative.

The effective action in the integral (2.1) is not gauge invariant. Contrary to the Abelian case the gauge invariance is broken not only by the gauge fixing term but also by the Faddeev-Popov ghost Lagrangian. To avoid this complication I propose the following construction.

Let us consider the path integral

$$S = \int \exp\left\{ i \int [L_{\mathrm{YM}} + (D_\mu\varphi)^*(D_\mu\varphi) - (D_\mu\chi)^*(D_\mu\chi) + (D_\mu b)^*(D_\mu e) + (D_\mu e)^*(D_\mu(b)]dx \right\} \delta(\partial_i A_i) d\mu' \tag{2.5}$$

The measure $d\mu'$ differs of $d\mu$ by the product of differentials of the scalar fields $(\varphi, \varphi^*, \chi, \chi^*, b, b^*, e, e^*)$.

We assume that the scalar fields comprise complex SU(2) doublets, the fields φ, χ are commuting and b, e are anticommuting. The integration goes over the scalar fields with radiation (Feynman) boundary conditions, which corresponds to considering the matrix elements between states which do not include excitations corresponding to the scalar ghost fields. The gauge fields in the integral (2.5) satisfy the boundary conditions

$$A_i^{\mathrm{tr}} \to A_i^{\mathrm{tr}}(in,out), \quad t \to \mp\infty \tag{2.6}$$

where A_i^{tr} are the three dimensionally transversal components of A_i, other fields having vacuum boundary conditions. Obviously due to the presence of $\delta(\partial_i A_i)$, $\partial_i A_i = 0$ at any time.

Performing explicitly the integration over the scalar fields in the eq. (2.5), we get the factor $(|D|^2)^{-2}$ from the integration over commuting fields φ and χ, and the factor $(|D|^2)^2$ from the integration over the anticommuting fields b and e. Hence the integral (2.5) coincides with the integral (2.1), which justifies the using in the l.h.s. of eq. (2.5) the same symbol S.

Let us make in the integral (2.5) the shift of the integration variables

$$\varphi \to \varphi + g^{-1}\hat{a}, \quad \chi \to \chi - g^{-1}\hat{a}, \quad \hat{a} = \left(0, \frac{a}{\sqrt{2}}\right) \tag{2.7}$$

a is a constant parameter. Instead of the eq. (2.5) now we have

$$\begin{aligned}\tilde{S} = \int \exp\Big\{ \int [&L_{\mathrm{YM}} + (D_\mu\varphi)^*(D_\mu\varphi) + g^{-1}(D_\mu\varphi)^*(D_\mu\hat{a}) \\ &+g^{-1}(D_\mu\hat{a})^*(D_\mu\varphi) + g^{-1}(D_\mu\chi)^*(D_\mu\hat{a}) + g^{-1}(D_\mu\hat{a}^*)(D_\mu\chi) + \\ &-(D_\mu\chi)^*(D_\mu\chi) + (D_\mu b)^*(D_\mu e) + (D_\mu e)^*(D_\mu b)]dx \Big\} \delta(\partial_i A_i) d\mu'\end{aligned} \tag{2.8}$$

Although at first sight the transformation (2.7) may influence the asymptotic behavior of the integration variables and therefore change the value of the integral, in our case it does not happen. Indeed making in the integral (2.8) the transformation

$$\begin{aligned}\varphi(x) &= \varphi'(x) - g^{-1} \int D^{-2}(x,y)(D^2\hat{a})(y)dy \\ \chi(x) &= \chi'(x) + g^{-1} \int D^{-2}(x,y)(D^2\hat{a})(y)dy\end{aligned} \tag{2.9}$$

which is a legitimate change of variables as $D^2\hat{a}$ is decreasing fast at $|t| \to \infty$, we are coming back to the eq. (2.5). (Note that it is impossible to integrate in the eq. (2.9) by parts, as $\hat{a}$ is a constant spinor.)

Notice that the choice of the negative sign of the kinetic term for the χ field is crucial for our construction. Due to the different signs of kinetic terms for φ and χ fields the shift (2.7) does not generate a mass term for the Yang-Mills field and preserves the equivalence of the modified theory to the original Yang-Mills model.

The action in the exponent (2.8) is invariant with respect to "shifted" gauge transformations

$$\begin{aligned}\delta A_\mu^a &= \partial_\mu \eta^a - g\varepsilon^{abc} A_\mu^b \eta^c \\ \delta\varphi^0 &= \frac{g}{2}\varphi^a \eta^a\end{aligned}$$

$$\begin{aligned}
\delta\varphi^a &= -\frac{a\eta^a}{2} - \frac{g}{2}\varepsilon^{abc}\varphi^b\eta^c - \frac{g}{2}\varphi^0\eta^a \\
\delta\chi^a &= \frac{a\eta^a}{2} - \frac{g}{2}\varepsilon^{abc}\chi^b\eta^c - \frac{g}{2}\chi^0\eta^a \\
\delta\chi^0 &= \frac{g}{2}\chi^a\eta^a \\
\delta b^a &= -\frac{g}{2}\varepsilon^{adc}b^d\eta^c - \frac{g}{2}b^0\eta^a \\
\delta b^0 &= \frac{g}{2}b^a\eta^a \\
\delta e^a &= -\frac{g}{2}\varepsilon^{adc}e^d\eta^c \\
\delta e^0 &= \frac{g}{2}e^a\eta^a,
\end{aligned} \tag{2.10}$$

where we introduced the representations of the scalar fields in terms of Hermitian components, e.g.

$$\varphi = \left(\frac{i\varphi_1 + \varphi_2}{\sqrt{2}}, \frac{\varphi_0 - i\varphi_3}{\sqrt{2}}\right) \tag{2.11}$$

This action is also invariant with respect to the supersymmetry transformations

$$\begin{aligned}
\delta\varphi(x) &= \epsilon b(x) \\
\delta\chi(x) &= -\epsilon b(x) \\
\delta e(x) &= \epsilon(\varphi(x) + \chi(x)) \\
\delta b &= 0
\end{aligned} \tag{2.12}$$

where ϵ is an anticommuting constant parameter. This invariance is closely related to the fact that the integral (2.5) in the sector which does not contain excitations corresponding to the scalar fields coincides with the Yang-Mills scattering matrix (see [10−12]).

Our proof of equivalence of representations (2.5) and (2.8) did not take into account a necessity of renormalization. One can see that to remove ultraviolet infinities generated in the perturbative expansion of the integral (2.8) mass renormalization of the type

$$\delta m_g(e^*b + b^*e), \quad \delta m_\varphi \varphi^*\varphi, \quad \delta m_\chi \chi^*\chi \tag{2.13}$$

may be needed, as well as new four point vertices

$$\gamma(e^*b + b^*e)^2, \quad \mu(\varphi^*\varphi)^2, \quad \varrho(\chi^*\chi)^2 \tag{2.14}$$

A possible counterterm structures not present in the Lagrangian (2.8) and compatible with the symmetries (2.10), (2.12) are

$$\begin{aligned}
&A[b^*e + e^*b + \varphi^*\varphi - \chi^*\chi + a(\varphi_0 + \chi_0)] \\
&B[b^*e + e^*b + \varphi^*\varphi - \chi^*\chi + a(\varphi_0 + \chi_0)]^2
\end{aligned} \tag{2.15}$$

where $A \sim g^2$, $B \sim g^4$. Note that the terms linear in the fields φ, χ are present in the eq. (2.15), corresponding to the necessity of the tadpole renormalization. In general any counterterms compatible with the symmetries of the theory may arise. In the presence of such terms in the exponent of the integral (2.8) one cannot any more to prove the equivalence of this expression to the original representation (2.5) for the Yang-Mills scattering matrix by simple shift of integration variables. Nevertheless the invariance of the action in the exponent (2.8) with respect to the transformations (2.10), (2.12) provides the unitarity of the S-matrix (2.8) in the physical subspace which includes only transversal spin one excitations. The proof goes in analogy with the construction given in the papers ([10–12]).

The invariance of the action with respect to the supersymmetry transformations (2.12) leads to existence of the conserved charge Q and one can separate the physical subspace by requiring its annihilation by the charge Q. For asymptotic states we shall have

$$Q^0|\psi\rangle^{\rm as}_{\rm ph} = 0 \tag{2.16}$$

where Q^0 is the asymptotic conserved charge. The asymptotic charges has a form

$$Q^0 \sim \int [\partial_0 b^\alpha(\varphi+\chi)^\alpha - \partial_0(\varphi+\chi)^\alpha b^\alpha]d^3x \tag{2.17}$$

I recall that we are working in perturbation theory and assume that the interaction is asymptotically turned off. That means all the terms $\sim g$ do not contribute to the asymptotic charge. Being written in terms of creation and annihilation operators the asymptotic charge looks as follows

$$Q^0 \sim \int [a_b^{\alpha+}(a_\chi^{\alpha-}+a_\varphi^{\alpha-}) + (a_\chi^{\alpha+}+a_\varphi^{\alpha+})a_b^{\alpha-}]d^3k \tag{2.18}$$

where the operators $a^\pm$ satisfy the following (anti)commutation relations

$$\begin{aligned}
a_b^{\alpha-}(k)a_e^{\beta+}(k') + a_e^{\beta+}(k')a_b^{\alpha-}(k) &= \delta^{\alpha\beta}\delta(k-k')\\
a_e^{\alpha-}(k)a_b^{\beta+}(k') + a_b^{\beta+}(k')a_e^{\alpha-}(k) &= \delta^{\alpha\beta}\delta(k-k') \qquad (2.19)\\
a_\varphi^{\alpha-}(k)a_\varphi^{\beta+}(k') - a_\varphi^{\beta+}(k')a_\varphi^{\alpha-}(k) &= \delta^{\alpha\beta}\delta(k-k')\\
a_\chi^{\alpha-}(k)a_\chi^{\beta+}(k') - a_\chi^{\beta+}(k')a_\chi^{\alpha-}(k) &= -\delta^{\alpha\beta}\delta(k-k') \qquad (2.20)
\end{aligned}$$

The operator Q^0 is obviously nilpotent as the operators a_b^+, a_b^- are anticommuting and the operators $(a_\chi^- + a_\varphi^-), (a_\chi^+ + a_\varphi^+)$ are mutually commuting.

Nonnegativity of the subspace annihilated by the operator Q^0 may be proven in the usual way (see [13]). Introducing the number operator for unphysical scalar modes

$$\hat{N} = \int \{a_\varphi^+(\mathbf{k})a_\varphi^-(\mathbf{k}) - a_\chi^+(\mathbf{k})a_\chi^-((\mathbf{k}) + a_b^+(\mathbf{k})a_e^-(\mathbf{k}) + a_e^+(\mathbf{k})a_b^-(\mathbf{k})\}d^3k \tag{2.21}$$

we see that this operator may be presented as the anticommutator

$$\hat{N} = [Q^0, K^0]_+ \tag{2.22}$$

where

$$K^0 = \int \{a_e^+(\mathbf{k})(a_\chi^-(\mathbf{k}) - a_\varphi^-(\mathbf{k})) + (a_\chi^+(\mathbf{k}) - a_\varphi^+(\mathbf{k}))a_e^-(\mathbf{k})\}d^3k \tag{2.23}$$

Applying the number operator (2.21) to an arbitrary vector we get

$$\hat{N}|\psi\rangle = N|\psi\rangle \tag{2.24}$$

if $N \neq 0$ it follows that

$$N|\psi\rangle = Q^0 K^0|\psi\rangle + K^0 Q^0|\psi\rangle \tag{2.25}$$

and any vector annihilated by Q^0 has a form

$$|\tilde{\psi}\rangle = |\psi\rangle_{A,c} + Q^0|\omega\rangle \tag{2.26}$$

where $|\psi\rangle_{A,c}$ does not contain the excitations corresponding to the ghost fields φ, χ, b, e. Recollecting that this vector contains only three dimensionally transversal excitations of the Yang-Mills field we conclude that

$$|\psi\rangle_{\rm ph}^{\rm as} = |\psi\rangle_{\rm tr} + |N\rangle \tag{2.27}$$

Here the vector $|\psi\rangle_{\rm tr}$ depends only on the three dimensionally transversal Yang-Mills field excitations and $|N\rangle$ is a zero norm vector orthogonal to $|\psi\rangle_{\rm tr}$. Factorizing this subspace with respect to the vectors $|N\rangle$ we see that the S-matrix (2.8) with the counterterms respecting the gauge invariance (2.10) and supersymmetry (2.12) is unitary in the subspace which contains only three dimensionally transversal excitations of the Yang-Mills field.

3. An unambiguous Lorentz covariant gauge with gauge invariant ghost interaction

Up to now we considered the Yang-Mills theory in the Coulomb gauge and our reformulation did not give any advantages in comparison with the standard one. In particular non gauge invariant interaction of the Faddeev-Popov ghosts was present. However we may pass in the integral (2.8) to some other gauge and get rid off the non gauge invariant ghost field Lagrangian. The new gauge condition avoids the problem of existence of Gribov copies for large fields.

We consider the gauge

$$\varphi^a - \chi^a = 0 \tag{3.1}$$

Obviously this condition selects a unique representative in the class of gauge equivalent configurations.

To pass to this gauge we shall use the standard Faddeev-Popov trick, multiplying the integral (2.8) by "one"

$$\Delta \prod_a \int \delta(\varphi^\Omega - \chi^\Omega)^a d\Omega \tag{3.2}$$

At the surface $\varphi^a - \chi^a = 0$ the gauge invariant functional Δ is equal to

$$\Delta^{-1} = \prod_x \left(a + \frac{g}{2}(\varphi^0 - \chi^0)\right)^{-3} \tag{3.3}$$

Hence in the gauge (3.1) the S-matrix generating functional may be written as follows

$$S = \int \exp\Big\{ i \int [L_{\rm YM} + (D_\mu\varphi)^*(D_\mu\varphi) - (D_\mu\chi)^*(D_\mu\chi) \\ + g^{-1}[(D_\mu\varphi)^* + (D_\mu\chi)^*](D_\mu\hat{a}) + g^{-1}(D_\mu\hat{a})^*(D_\mu\varphi + D_\mu\chi) + (D_\mu b)^*(D_\mu e) \\ + (D_\mu e)^*(D_\mu b) + \lambda^a(\varphi^a - \chi^a)]dx \Big\} \Delta d\tilde{\mu} \tag{3.4}$$

The measure $d\tilde{\mu}$ is the product of all the fields differentials and does not include any dynamical ghost determinants. All the terms in the exponent except for the gauge fixing term $\int \lambda^a(\varphi^a - \chi^a)dx$ are invariant with respect to the gauge transformations (2.10).

In the same way one can introduce a nonsingular α gauge, changing in the eq. (3.2) $\delta(\varphi^\Omega - \chi^\Omega)$ by $\delta[(\varphi^\Omega - \chi^\Omega)^a - c^a(x)]$ with arbitrary function $c^a(x)$ and then integrating over c^a with the weight $\exp\{\frac{i}{2\alpha}\int c^2(x)dx\}$. The ghost structure of the effective Lagrangian remains the same and the condition $\varphi^a - \chi^a = c^a$ also selects a unique representative in the class of the gauge equivalent configurations. However the effective Lagrangian in this gauge is more complicated as it contains vertices including the field $\varphi^a - \chi^a$ which disappear in the gauge used in this paper. In $\alpha = 0$ gauge which we are using, the part of the effective Lagrangian which describes the interaction of bosonic scalars with the Yang-Mills field looks as follows

$$L_1 = \partial_\mu\varphi^{0+}\partial_\mu\varphi^{0-} + a\partial_\mu\varphi^{a+}A_\mu^{a+} + \frac{g^2}{8}A_\mu^2(\varphi^{0+}\varphi^{0-}) \\ + \frac{ag^2}{4}A_\mu^2\varphi^{0+} - \frac{g}{2}\partial_\mu\varphi^{0-}\varphi^{a+}A_\mu^a + \frac{g}{2}\varphi^{0-}\partial_\mu\varphi^{a+}A_\mu^a \tag{3.5}$$

where we used natural notations $\varphi^{\alpha\pm} = \varphi^\alpha \pm \chi^\alpha$.

The integral (3.4) includes a local measure, which may be formally presented as an addition to the action having a form

$$\delta A = \int \delta^4(0) \ln\left(1 + \frac{g(\varphi^0 - \chi^0)^3}{2a}\right) d^4x \tag{3.6}$$

This term compensates some ultraviolet divergencies present in the diagrams generated by the expansion of the integral (3.4). We shall not analyze this cancelation in details and assume that this integral is calculated by using a regularization similar to the dimensional one, that is we omit all counterterms proportional to $\delta(0)$ or $D_c(0)$.

The free action determining the propagators for the perturbative expansion of the integral (3.4) looks as follows

$$A_0 = \int \Big[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2}\partial_\mu\varphi^0\partial_\mu\varphi^0 \\ - \frac{1}{2}\partial_\mu\chi^0\partial_\mu\chi^0 + a\partial_\mu\varphi^a A_\mu^a + \frac{1}{2}\partial_\mu b^\alpha\partial_\mu e^\alpha \Big] dx \tag{3.7}$$

where we used the gauge condition $\varphi^a = \chi^a$.

One sees that the propagators $\varphi^0, \varphi^0; \chi^0, \chi^0; b^\alpha, e^\beta; A^{\rm tr}_\mu, A^{\rm tr}_\nu$ have a standard form and for large k decrease as k^{-2}, whereas the mixed propagator $\varphi^a, \partial_\mu A^a_\mu$ is a constant $\sim a^{-1}$. The free field A_μ satisfies the condition $\partial_\mu A^a_\mu = 0$.

Account of the interaction leads to modification of this condition. Variation of the Lagrangian (3.5) with respect to φ^a leads to the following condition on the interacting field A_μ

$$\left(a + \frac{g}{2}(\varphi^0 - \chi^0)\right) \partial_\mu A^a_\mu = -g A^a_\mu (\partial_\mu \varphi^0 - \partial_\mu \chi^0) \tag{3.8}$$

Renormalization of our model is not quite trivial, because one of the propagators determined by the free Lagrangian (3.7) does not decrease at infinity sufficiently fast: $\varphi^a, A^b_\mu \sim k^{-1}$. One may expect that it will lead to nonrenormalizability. However the fields φ^a with slowly decreasing propagators are always accompanied by the fields φ^{0-},which have a nonzero contraction only with φ^{0+}. The field φ^{0+} enters a superrenormalizable vertex $A^2_\mu \varphi^{0+}$. Contrary to nonrenormalizable models, in our case the degree of divergency of arbitrary diagram is limited. In particulat the degree of divergency of any diagram with only Yang-Mills field external lines is the same as in the standard formulation.

Now we shall derive the relations between the Green functions, which follow from the gauge invariance of the effective action in the eq. (3.4) and replace the usual ST identities in the present case.

We consider the Green function generating functional given by the integral

$$Z = \int \exp\left\{ i \int [\tilde{L}(A_\mu, \varphi, \chi, b, e) + \lambda^a(\varphi^a - \chi^a) \right.$$
$$\left. + J^a_\mu A^a_\mu + \zeta^\alpha(\varphi^\alpha - \chi^\alpha) + \xi^\alpha(\varphi^\alpha + \chi^\alpha) + \kappa^* b + b^* \kappa + \sigma^* e + e^* \sigma] dx \right\} d\mu \tag{3.9}$$

where $\tilde{L}$ is the gauge invariant Lagrangian standing in the exponent of the integral (3.4), and $J_\mu, \zeta, \xi, \kappa, \sigma$ are external sources. Let us make the change of variables given by the eq. (2.10). Due to the gauge invariance of the Lagrangian $\tilde{L}$ the only terms which change under this transformation are the source terms and the gauge fixing term. Using the fact that the integral(3.9) does not change under this transformation we get

$$\int \exp\left\{ i \int [\tilde{L} + \lambda^a(\varphi^a - \chi^a) + s.t.] dx \{ \lambda^a(y) \left[a + \frac{g}{2}(\varphi^0(y) - \chi^0(y)) \right] \right.$$
$$\left. + i\partial_\mu J^a_\mu(y) + \zeta^a(y) \left[a + \frac{g}{2}\left(\varphi^0(y) - \chi^0(y) \right) \right] + \cdots \right\} d\mu = 0 \tag{3.10}$$

Here $s.t.$ stands for the source terms and $\ldots$ denote the variation of all remaining source terms. It is convenient to make further redefinitions:

$$\lambda^a \left(a + \frac{g}{2}(\varphi^0 - \chi^0) \right) = \lambda'^a \tag{3.11}$$

$$\varphi^a - \chi^a = (\varphi^a - \chi^a)' \left(a + \frac{g}{2}(\varphi^0 - \chi^0) \right) \tag{3.12}$$

After such redefinition the eq. (3.10) acquires the form

$$\int \exp\left\{i\int\left[\tilde{L}(A_\mu,(\varphi^a+\chi^a),(\varphi^a-\chi^a)\left(a+\frac{g}{2}(\varphi^0-\chi^0),\varphi^0,\chi^0,b^\alpha,e^\alpha\right)\right.\right.$$
$$\left.\left.+\lambda^a(\varphi^a-\chi^a)+J^a_\mu A^a_\mu+\zeta^a(\varphi^a-\chi^a)\left(a+\frac{g}{2}(\varphi^0-\chi^0)\right)+\cdots\right]dx\right\}$$
$$\left\{\lambda^a(y)+\partial_\mu J^a_\mu(y)+\zeta^a(y)\left(a+\frac{g}{2}(\varphi^0(y)-\chi^0(y)\right)+\cdots\right\}d\mu=0 \tag{3.13}$$

Here ... denote all remaining source terms and their variations under transformation (2.10).

This equation replaces the standard system of ST identities. In particular the simplest identities, which follow from the eq. (3.13) are

$$\langle\lambda^a(x)A^b_\mu(y)\rangle = \partial_\mu\delta(x-y)\delta^{ab} \tag{3.14}$$
$$\langle\lambda^a(x)(\varphi+\chi)^b\rangle = 0 \tag{3.15}$$

We postpone a detailed analysis of these relations as well as consideration of the renormalization procedure to a separate publication.

As follows from our previous discussion the S-matrix defined by the eq. (3.4) coincides with the Coulomb gauge scattering matrix given by the eq. (2.1). The eq. (2.1) strictly speaking defines a unique S-matrix for the Yang-Mills theory only in perturbation theory. Although the eq. (3.4) formally makes sense beyond the perturbation theory, the proof of unitarity relies on its equality to the Coulomb gauge S-matrix. At present I do not know an independent proof of the unitarity of the S-matrix (3.4).

4. Discussion

The main goal of this paper was to show that Yang-Mils theory allows a manifestly Lorentz invariant formulation with gauge invariant ghost fields interaction. In this formulation Yang-Mills theory demonstrates a remarkable similarity to QED. In particular as in QED the gauge condition (3.4) does not lead to an ambiguity in the choice of representative in the class of gauge equivalent configurations. The relations between the Green functions which replace in this case the standard ST-identities also may be derived in a way similar to QED. Gauge invariance of the effective action simplifies the construction of invariant regularization and may be helpful for invariant regularization of non-Abelian supersymmetric models. Finally this construction may be useful for a nonperturbative analysis of the Green functions on the basis of Dyson-Schwinger equations having in mind that the gauge condition (3.1) does not introduce Gribov ambiguity. This problem requires further investigation.

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